

Due February 7

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 Name

Remember, you are not to discuss these problems with anyone with three exceptions: (1) discussions with me are allowed, (2) you may use any information that comes to light during a Wednesday Brainstorming session and (3) if the directions to the problem specifies you may work with others.

*“It is by logic that we prove but by intuition that we discover.”* (Henri Poincaré)

**Undefined terms:** point, line, incident with

**Incidence Axiom 1**

For every point  $P$  and for every point  $Q$  not equal to  $P$  there exists a unique line  $l$  incident with  $P$  and  $Q$ .

**Incidence Axiom 2**

For every line  $l$  there exist at least two distinct points incident with  $l$ .

**Incidence Axiom 3.**

There exist three distinct points with the property that no line is incident with all three of them.

**Problems**

- Be ready to answer the Review Exercise Questions in Chapter 1 and Chapter 2.

I. Prove the Proposition(s) from the following list assigned to you in class. (2,3,4,5,7)

**Proposition 1 (2.1)** *(Not Assigned) If  $l$  and  $m$  are distinct lines that are not parallel, then  $l$  and  $m$  have a unique point in common*

**Proposition 2 (2.2)** *There exist three distinct lines that are not concurrent.*

**Proposition 3 (2.3)** *For every line there is at least one point not lying on it.*

**Proposition 4 (2.4)** *For every point there is at least one line not passing through it.*

**Proposition 5 (2.5)** *For every point  $P$  there exist at least two lines through  $P$ .*

**Proposition 6 (2.6)** *For every point  $P$  there are at least two distinct points neither of which is  $P$ .*

**Proposition 7 (2.7)** *For every line  $l$  there are at least two distinct lines neither of which is  $l$ .*

**Proposition 8 (2.8)** *If  $l$  is a line and  $P$  is a point not incident with  $l$  then there is a one-to-one correspondence between the set of points incident with  $l$  and the set of lines through  $P$  that meet  $l$ .*

**Proposition 9 (2.9)** *Let  $P$  be a point. Denote the set of points  $\{X : X \text{ is on a line passing through } P\}$  by  $S$ . Then  $S$  contains every point.*

**Proposition 10 (2.10)** *Let  $l$  be a line. Denote the set of lines  $\{m : m \text{ is incident with a point that lies on } l \text{ or } m \text{ by } L$ . Then  $L$  contains every line.*

II. Do the problems from the following list that were assigned to you in class.

1. Show the interpretation in Example 3 of the text is a model of Incidence geometry. Further, show this model satisfies the Euclidean parallel property.

2. Show the interpretation in Example 4 of the text is a model of Incidence geometry. Further, show this model satisfies the hyperbolic parallel property.
3. Construct an interpretation of Incidence geometry in which Incidence Axioms 1 and 2 hold but 3 fails. Explain why problems II.2, II.3, and II.4 together show that it is impossible to prove any of the three axioms using only the other two.
4. Construct an interpretation of Incidence geometry in which Incidence Axioms 2 and 3 hold but 1 fails. Explain why problems II.2, II.3, and II.4 together show that it is impossible to prove any of the three axioms using only the other two.
5. Construct an interpretation of Incidence geometry in which Incidence Axioms 1 and 3 hold but 2 fails. Explain why problems II.2, II.3, and II.4 together show that it is impossible to prove any of the three axioms using only the other two.

**III.** A number of the interpretations in this exercise will be seen later in the semester. Be sure to familiarize yourself with all of them.

Do the problem in the following list assigned to you in class.

1. Exercise 9.a
2. Exercise 9.b
3. Exercise 9.c
4. Exercise 9.d
5. Exercise 9.e

**IV.** Do the problem in the following list assigned to you in class.

1. Exercise 10.a
2. Exercise 10.b
3. Exercise 10.c
4. Exercise 11.
5. Exercise 13. [Devlin's book might be useful in your justifications.]

1. Exercise 2 of Chapter 2.
2. Exercise 3 of Chapter 2. Negate the Euclidean Parallel Postulate (Page 19).
3. Exercise 4 of Chapter 2. Negate Euclid's fourth postulate (Page 18).
4. Exercise 5 of Chapter.